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Research on Competitive Location of Restaurants Based on Utility Theory

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ABSTRACT

For a chain restaurant planning to expand its business, the primary challenge lies in focusing on how to determine the optimal location for a new restaurant through the optimization of site selection methods. For a better analysis, this paper establishes the Maximum Market Share Model (Model I), with model elements decomposed into population distribution, purchasing power, and consumption habits. The Logit utility function representing consumer choice behavior is incorporated into the site selection model, establishing the Maximum Market Coverage Model based on competitors (Model II). Sensitivity analysis indicates that purchasing power and distance factors in consumption habits are more sensitive than other factors. Stability analysis shows that the improved iterative algorithm converges quickly and produces highly stable results.

Keywords: Restaurant Site Selection; Logit Utility Function; Market Competition

1 INTRODUCTION

As people's purchasing power increases, chain operation has become the most important business model in developed countries. In recent years, with the wave of global economic integration, some developed countries' chain enterprises, leveraging their abundant capital, mature technology, and excellent management, have expanded their store openings worldwide, forming massive chain systems [1].

The region A, with a population of 500,000, has three similar chain restaurants. One is located in the business district and has performed exceptionally well, while the other two are situated in the city center and the suburbs, with smaller impact ranges.

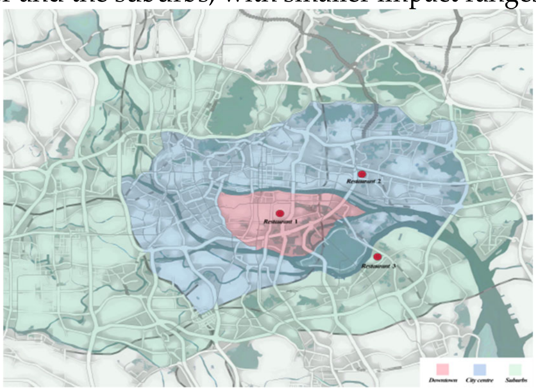

Figure 1: Map of Area A

2 MODEL FOR MAXIMUM MARKET COVERAGE

2.1 Population density

Clark(1979) model posits that in a certain region, population is more concentrated in central areas, gradually decreasing as distance from the central location increases.

Consequently, the population density diminishes with increasing distance from the central position [2]. Therefore, the density of a specific point in a demand region can be expressed as:

$$D_{(x_i, y_i)} = D_{(\overline{X}, \overline{Y})} \exp(-\alpha d_{ij}) \tag{1}$$

To calculate the population density at the centroid coordinate position based on the method proposed by Jin Jun (2003) and others:

$$D_{(\overline{X}, \overline{Y})} = \frac{Num_i}{\sum_{j=1}^{n} (f_{ij} \cdot S_{ij})} \tag{2}$$

2.2 Purchasing power

Purchasing power can be expressed as:

$$B_i(X) = e_i D(x_i, y_i) \tag{3}$$

2.3 Establishment of the model

Thus, we can represent consumers choosing the restaurant location as 1, simplifying the mathematical expression of the model's objective function to:

$$\max\ Z = \sum_{i=1}^{n} B_i(x) = \sum_{i=1}^{n} \left(D_{(\overline{X}, \overline{Y})} e_i \cdot e^{-\alpha d_{ij}} \right) \tag{4}$$

The constraint conditions are:

$$d_{ij} = \sqrt{(x_i - X_j)(y_i - Y_j)} \quad \forall i \in I, \forall j \in J \tag{5}$$

$$x \in [x_l, x_r] \tag{6}$$

$$y \in [y_r, y_l] \tag{7}$$

Among them, the constraint conditions (6) and (7) respectively indicate that the location for choosing to establish a new restaurant must be within region A [3].

3 MAXIMUM MARKET COVERAGE MODEL BASED ON COMPETITORS

3.1 Utility function

The utility function representing the patronage of restaurant *j* by consumers at demand point *i* is expressed as:

$$U_{ij} = u_{ij} + \varepsilon_{ij} = u(h_{ij}, b_i, d_{ij}, \theta) + \varepsilon_{ij} \tag{8}$$

Therefore, we can rewrite the above expression as:

$$U = F(x_1, x_2, \ldots, x_n) \tag{9}$$

Considering that each indicator factor x_i is mutually independent, when each indicator factor has a different impact on consumer choices, it is necessary to assign different weights to each factor. That is,

$$U = \sum_{i=1}^{n} w_i f_i(x_i) \tag{10}$$

In this case, the utility function for a particular restaurant can be defined as:

$$U(Q, d) = \sum_{i=1}^{N} w_i Q_i + e^{-\beta d} \tag{11}$$

The utility value is a positive number ranging between 0 and 1.

Assuming that each consumer's evaluation of the utility of a restaurant is independent of others, we can derive the mean and variance of the utility:

$$E[U(Q,d)] = E\left[e^{-\beta d}\right] + E[Q] \tag{12}$$

$$Var[U(Q,d)] = Var\left[e^{-\beta d}\right] + Var[Q] \tag{13}$$

When selecting a new restaurant location, apart from the unknown distance *d* from the demand area, the other utility influencing factor *Q* is known. In other words, the Logit utility function is expressed as:

$$P_i = P\{U_i > U_j, j \neq i\} = \int_{e^i} F\{u_i - u_j + \varepsilon_i, j \neq i\} f_i(y)dy \tag{14}$$

The random disturbance term ε_j is an independently and identically distributed random variable, following the Gumbel distribution. Its probability density function (PDF) and cumulative distribution function (CDF) are as follows:

$$f(\varepsilon_j) = e^{-\varepsilon_j} \cdot e^{-e^{-\varepsilon_j}} \tag{15}$$

$$F(\varepsilon_j) = e^{-e^{-\varepsilon_j}} \tag{16}$$

As the random disturbance term ε_j is independent, the joint distribution of $\forall j \neq i$ and ε_j is the product of the distributions for each ε_j, i.e.,

$$P_i \mid \varepsilon_i = \prod_{j \neq i} e^{-e^{-(u_i - u_j + e_i)}} \tag{17}$$

Considering that ε_j is random, so

$$P_i = \int_{\varepsilon_i = -\infty}^{+\infty} \exp[(-\exp(-\varepsilon)\sum_j \exp(-u_i + u_j)] \cdot \exp(-\varepsilon_i) d\varepsilon_i \tag{18}$$

Let $t = e^{-\varepsilon_i}$ undergo a change of variables in the above expression to obtain

$$P_i = \frac{e^{u_i}}{\sum_j e^{u_j}} (i \neq j) \tag{19}$$

Similarly, the selection of restaurant *i* by a specific demand area is inevitably influenced by other restaurants *j*. Therefore, we assume that the pairwise correlation coefficient for consumers' choices of restaurants is σ. According to the Logit model:

$$F(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) = \exp\left\{-\left[\sum \exp(-\varepsilon_j / \rho)\right]^{\rho}\right\} \tag{20}$$

We can rewrite the formula as:

$$P(y = i \mid g) = \frac{e^{\frac{u_i}{\rho}}}{\sum_g e^{\frac{u_i}{\rho}}} \tag{21}$$

$\rho \in [0,1]$, when $\rho = 1$, there is no similarity between any two alternative restaurants.

3.2 Simulating the impact of the distribution of restaurant populations

In other words, the probability that consumers in demand area *i* choose restaurant *j* is greater than the probability of choosing restaurant *l*.

$$\Pr\left[U(Q_j, d_{ij}) - U(Q_l, d_{il}) \geq 0\right] \tag{22}$$

According to formula (22), we can obtain:

$$P(d) = \Pr\left[Z \geq \frac{5(d-1)}{\sqrt{1+d^2}}\right] \tag{23}$$

By solving, we can obtain the probability of consumer choice when there is one competitor in a certain area, as shown in Figure 2.

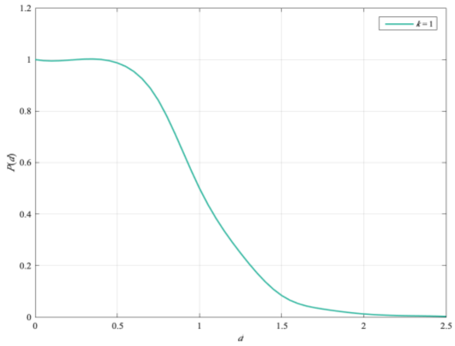

Figure 2: P (d) curve with k=1

Similarly, we set k to be 2 and 3 to simulate the impact of the distribution of restaurant populations in region A on the probability of consumer choices. The specific results are shown in Figure 3.

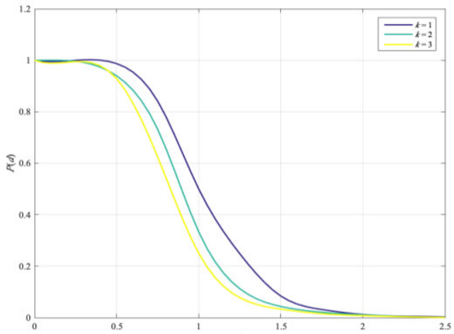

Figure 3: P (d) curve with k=1,2,3

From Figure 3, it is evident that for different population distribution scenarios, $P(d)$ all exhibits an S-shaped curve. To facilitate the computation of the model, we fit the function $P(d)$. We assume the fitted function to be:

$$p(d)=\frac{1}{1+e^{\gamma+\omega d+\lambda d^2}} \tag{24}$$

Considering that there are 3 competing restaurants in region A, we solve for $k = 3$, and the formula (24) can be written as:

$$p(d)=\frac{1}{1+e^{\gamma+\omega d+\lambda d^2}} \tag{25}$$

The fitting results are shown in Figure 4, and the specific parameters are shown in Table1

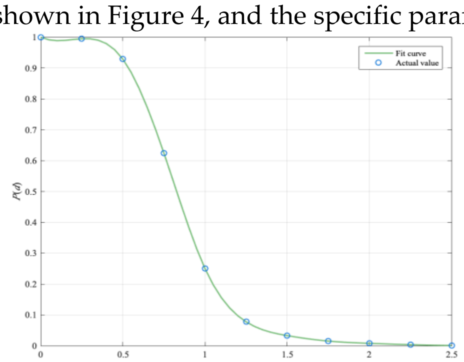

Figure 4: Fit curve

Table 1: Fitting parameter results

γ	ω	λ
-7.1379	10.5152	-2.2715

As shown in Figure 4, the effect of fitting the curve is very good, so this fitting function can be used as an alternative to formula (25).

3.3 Establishment of the model

When there are competitors in the area, consumers can choose a restaurant based on their preferences. In this case, we need to consider the probability of consumers' choices [4-9]. Maximum Market Coverage Model with Competitors is obtained as follows:

$$\max Z = \sum_{i=1}^{n} B_i(x)P_i(d) = \sum_{i=1}^{n}\left(D_{(\overline{X},\overline{Y})} e_i \cdot e^{-\alpha d_{ij}} \cdot \frac{1}{1+e^{\gamma+\omega d_i+\lambda d_i^2}} \right) \tag{26}$$

The constraint conditions are:

$$d_{ij} = \sqrt{(x_i - X_j)(y_i - Y_j)} \quad \forall i \in I, \forall j \in J \tag{27}$$

$$x \in [x_l, x_r], y \in [y_r, y_l], X \neq X_p$$

Considering the exclusion effect between competing restaurants, we have improved formula (28) to:

$$\max Z = \sum_{i=1}^{n} B_i(x)P_i(d) + \sum_{i=1}^{n} B_i(x)P_i(d) \times \sum_{k} \zeta_k d_{jk} \tag{28}$$

When $\zeta_k > 0$, it represents the aggregation effect; When $\zeta_k < 0$, it indicates exclusion effect, and the value of ζ_k can be set based on different types of restaurant competition.

4 MAXIMUM MARKET COVERAGE MODEL BASED ON COMPETITORS AND COSTS

Considering that different locations correspond to different housing prices and rents, resulting in different site selection costs, this article represents the site selection cost of a facility at a certain location as a function related to the location of the facility:

$$Cost(d) = \frac{5000}{a + bd + cd^3} \tag{29}$$

Using (P_x, P_y) to represent the coordinates of the location with the highest land price, (x, y) is the location of the restaurant, then formula (29) can be converted to:

$$Cost(X) = \frac{5000}{a + b\sqrt{(x-P_x)^2 + (y-P_y)^2} + c\sqrt[3]{(x-P_x)^2 + (y-P_y)^2}} \tag{30}$$

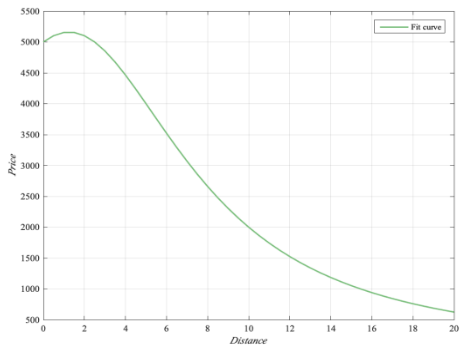

Figure 5: Rent fitting results

Table 2: Fitting parameter results

a	b	c
1	-0.05	0.02

Building upon the model of Question Two, we introduce the objective function into site selection costs, resulting in the mathematical expression of the objective function for the Maximum Market Coverage Model:

$$\max Z = 0.7 \times \sum_{i=1}^{n} \left(D_{(\overline{X},\overline{Y})} e_i \cdot e^{-\alpha d_{ij}} \cdot \frac{1}{1+e^{\gamma+\omega d_i+\lambda d_i^2}} \right) - Cost(X) \tag{31}$$

The constraint conditions are:

$$d_{ij} = \sqrt{(x_i - X_j)(y_i - Y_j)} \quad \forall i \in I, \forall j \in J \tag{32}$$

$$x \in [x_l, x_r], y \in [y_r, y_l], X \neq X_p$$

Among them, the constraint conditions (31) and (32) respectively indicate that the location for establishing a new restaurant must be within the range of area a. The constraint conditions (6-5) represent geographical limitations.

5 MARKET POTENTIAL MODEL

5.1 Consumption subject

The larger the total population of a region, the greater the market potential. As resources in a region are limited, population growth will slow down when it reaches a certain level [10]. At this point, we use the Logistic Growth Model to predict the future population changes in region A.

$$C(t) = \frac{C_{\max}}{1 + \left(\frac{C_{\max}}{C_0} - 1 \right) e^{-rt}} \tag{33}$$

5.2 Purchasing power

We introduce a time variable based on formula (33), and the formula becomes:

$$B(t) = e(t)D(t) \tag{34}$$

$$e(t) = e_0 \cdot \lambda^t \tag{35}$$

$$B(t) = e_0 \cdot \lambda^t \cdot D(t) \tag{36}$$

5.3 Consumer demand

Therefore, we use formula (10) to calculate the utility value of different cuisine restaurants for each population, that is, the demand level of different restaurants for different populations. At this point, the expression of the consumption demand function is

$$H = \sum_{i=1}^{n} w_i V_i \tag{37}$$

5.4 Establishment of the model

We combine economic development trends, analyze consumer subjects, purchasing power, and consumption demand, and ultimately establish a market potential model is

$$M(t) = C(t) \times B(t) \times H \tag{38}$$

6 MODEL SOLVING

6.1 Results for Problem 1

Table 3 lists the relevant hypothetical data of population distribution calculated based on statistical results and methods.

Table 3: About Data Information

Demand area *i*	X_i (km)	Y_i (km)	per capita consumption expenditure (USD/month)	Center density D_i (person/m^2)
1	10	10	500	0.2
2	12	10	450	0.15
3	16	14	400	0.08

Assuming the positions of three alternative restaurants within the site selection area, Table 4 shows the coordinates of the positions of the three alternative restaurants.

Table 4: Alternative restaurant information

Alternative restaurants *j*	X_i (*km*)	Y_i (*km*)
1	8	10
2	10	12
3	14	14

By solving, the optimal location is (8 *km*,8 *km*), which is the coordinate of alternative restaurant 1. As shown in Figure 6.

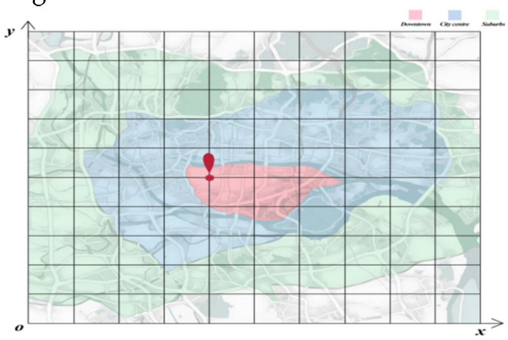

Figure 6: New restaurant site selection results

6.2 Results for Problem 2

Assuming there are 3 restaurants in Region A, and there is a competitive relationship between the newly added restaurants and the 3 restaurants, the specific location information is shown in Table 5. Please refer to Table 5 for relevant information on demand areas.

Table 5: Competitive Restaurant Coordinates

Existing restaurant restaurants *k*	X_i (*km*)	Y_i (*km*)
1	10	11
2	10	9.4
3	13.6	12

Write an iterative algorithm using Matlab, and the solution results are shown in Table 6. The convergence curve is shown in Figure 7.

Table 6: Solution results

Iterations	Coordinate
1	(16.000000 , 14.000000)
⋮	⋮
68	(11.278025 , 9.957387)

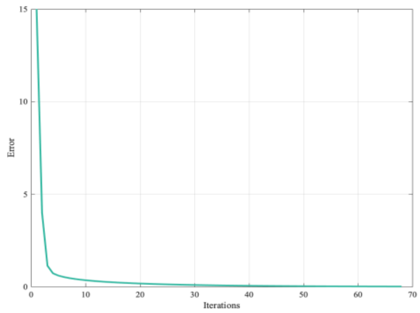

Figure 7: Convergence curve

From the results in Figure 7, it can be seen that after 40 iterations, the optimal site selection scheme converges to (11.28,9.96). The maximum market share is 578626.94.

6.3 Results for Problem 3

According to the results in Table 3, we can know that the rental cost function is:

$$Cost(X)=\frac{5000}{1-0.05\sqrt{(x-P_x)^2+(y-P_y)^2}+0.02\sqrt[3]{(x-P_x)^2+(y-P_y)^2}} \tag{39}$$

Based on Problem 2, we consider the cost function (45), and the results obtained through iterative algorithm are shown in Table 7. The convergence curve is shown in Figure 8.

Table 7: Solution results

Iterations	Coordinate
1	(16.000000 , 14.000000)
⋮	⋮
48	(11.299399 , 9.955890)

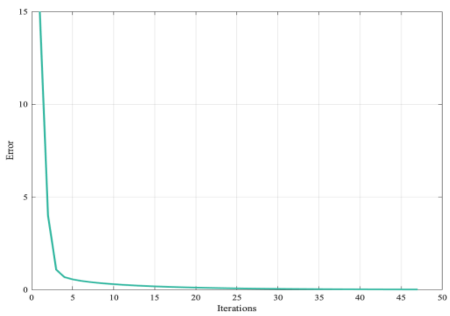

Figure 8: Convergence curve

From the results in Figure 8, it can be seen that after 30 iterations, the optimal site selection scheme converges to (11.30,9.96). The maximum market share is 352905.69.

6.4 Results for Problem 4

This article collects relevant data for analysis and fitting the parameters in the model, and finally predicts the market potential values of three regions (commercial centers, city centers, and suburbs) in Area A. The results are shown in Figure 9.

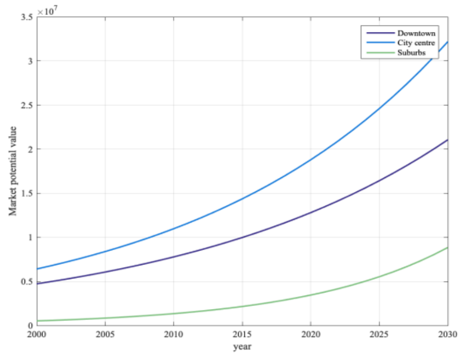

Figure 9: Market Potential Value Prediction Results

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