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Multi-Stage Network Optimization Model and Decomposition Algorithm for Improving the Efficiency of Enterprise Production System under the Background of Digital Economy

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Abstract: Driven by the digital economy, enterprise production systems feature multi-stage networking with deeply embedded data elements, posing challenges for traditional optimization methods in modeling coupling relationships and efficiency improvement. To address this, this paper constructs a multi-stage network optimization model (MSN-OPT) that integrates digital elements, abstracting the production system as a resource- and information-flow-driven directed network. The model maximizes overall system efficiency through collaborative optimization of resource allocation and data input, incorporating stage weights and digital input costs. Using Benders Decomposition and column generation, the high-dimensional coupling problem is transformed into an iterative master-subproblem framework, significantly reducing computational complexity. Numerical experiments on a 100-200 node production network show that the proposed method reduces computation time by 55%-67% while maintaining optimal solution consistency. Under various disturbance scenarios, system efficiency stabilizes within 12%-18%. Sensitivity analysis reveals that when digital element disturbances are controlled within ±10%, efficiency fluctuations remain below 3%, confirming the model's robustness and stability. This paper provides a computable modeling approach and efficient algorithmic support for production system optimization in the digital economy context.

Keywords: Digital economy; Multi-stage network optimization; Enterprise production system; Efficiency improvement; Decomposition algorithm

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1. INTRODUCTION

With the rapid development of digital economy, the operation mode of enterprise production system is undergoing profound changes. Digital elements, with data at their core, have gradually become important production factors alongside capital and labor, making the

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traditional production system dominated by physical resources turn to a new structure of "data-driven information collaboration and intelligent decision-making" [1],[2],[3]. In this context, the enterprise production process is no longer a single linear process, but a complex network system composed of multiple interrelated stages, which can realize dynamic interaction and collaborative optimization through data flow and resource flow between different stages. The introduction of digital technology not only changes the way of resource allocation, but also strengthens the coupling relationship between stages, making the production system show obvious characteristics of multi-stage, networking and dynamic evolution [4],[5]. Therefore, how to characterize this new production structure in the context of the digital economy and realize the overall improvement of system efficiency has become a key problem to be solved in the current theory and practice.

However, the optimization modeling of enterprise production system efficiency improvement faces many challenges. Firstly, the multi-stage structure brings complex coupling relationship, and there are resource transfer and information feedback between different stages, which makes it difficult to decompose the system into independent subproblems; Secondly, digital elements involve significant uncertainty, and their effects depend on data quality, processing capacity, and fluctuations in the external environment, which increases the complexity of model construction and solution; Thirdly, with the expansion of enterprise scale and the complexity of production network, the number of decision variables and constraints increases rapidly, which leads to the high-dimensional and large-scale characteristics of optimization problems, and the traditional methods are difficult to obtain high-quality solutions in an acceptable time. these factors jointly constrain modeling accuracy and solution efficiency of the efficiency improvement problem.

The existing research provides a theoretical foundation for enterprise efficiency analysis to some extent, but there are still obvious deficiencies. On the one hand, many methods are based on simplified single-stage or weak coupling structure, which is difficult to accurately reflect the characteristics of multi-stage network under the condition of digital economy; on the other hand, although some models consider complex structures, they lack support from efficient algorithms, which is difficult to deal with large-scale problems in practical applications. In addition, most studies on the mechanisms of digital elements remain at the level of empirical analysis, lacking systematic methods to internalize them into the optimization model [6]. Therefore, it is necessary to carry out systematic innovation from two aspects of model structure and algorithm design to make up for the deficiencies of existing research.

Based on the above background, this paper focuses on improving the efficiency of enterprise production systems in the digital economy environment, aims to build an optimization model that can describe the multi-stage network structure and the interaction mechanism of digital elements, and on this basis, designs an efficient decomposition algorithm to achieve the solvability of large-scale problems [7],[8]. Specifically, this paper models the enterprise production system as a multi-stage network structure, introduces the coupling relationship between data-driven production mechanism and information flow, establishes an optimization model for efficiency improvement, and uses the decomposition idea to split the complex problem into solvable sub problems, so as to significantly improve the computational efficiency while ensuring the accuracy of the solution.

The main innovations of this paper are as follows. First, at the structural level, the enterprise production system is modeled as a multi-stage network, which systematically depicts the coupling relationship between resource flow and data flow, breaking through the limitations of traditional single-stage modeling; Secondly, digital elements and their dynamic transmission mechanism are explicitly introduced into the model, so that the data-driven effect can be endogenous reflected in the optimization process; Thirdly, according to the high-dimensional and strong coupling characteristics of the model, an efficient algorithm based on decomposition idea is designed, which effectively improves the computational performance of large-scale problems; Finally, through the numerical experiments and sensitivity analysis of the system, the superiority of the model in efficiency improvement and algorithm performance is verified. The above innovations provide new theoretical tools and method support for the optimization of enterprise production system under the background of digital economy.

2. PROBLEM FORMULATION &MODEL

Under the background of digital economy, enterprise production system is no longer a single-stage input-output structure in the traditional sense, but a complex network system composed of multiple mutually coupled and data-driven production stages. To characterize this feature, this paper abstracts the enterprise production system as a directed network $G = (V, E)$, where the node set $V = \{1,2,\dots,K\}$ represents different stages of the production process (such as data acquisition, data processing, intelligent manufacturing and market transformation, etc.), and the edge set $E \subseteq V \times V$ represents the resource flow and information flow transmission relationship between stages. Each node not only undertakes the traditional physical production function, but also embeds the decision-making mechanism driven by digital elements, thus forming a "physical flow data flow" dual coupling structure. Different from the traditional network optimization model, this structure explicitly depicts the feedback and amplification effect of information on production efficiency under the condition of digital economy, which makes the system show obvious asymmetry and cross stage dependence.

In the above network structure, two types of core input elements are introduced: one is the traditional resource input x_i, where $i = 1,2,\dots,I$, including capital, labor and equipment; The other is digital element input z_j, where $j = 1,2,\dots,J$, which is used to characterize data resources, information system input and platform capability. The system produces the output y_k at each stage, where $k \in V$ represents the phased output or final product. In order to describe the characteristics of digital economy, this paper assumes that digital elements not only directly affect the level of output, but also transfer between different stages through information flow, thus changing the production function of subsequent stages [9],[10]. Specifically, in stage k, the output function can be expressed as:

$$y_k = f_k\left(x^k, z^k, s^k\right) \tag{1}$$

Where x^k and z^k respectively represent the resource input and digital input in stage k, and s^k represents the information state variables from the previous stage, which are used to describe the accumulation and transmission effect of data in the network. This setting enables the model to reflect the core mechanism of "data-enabled production ".

At the decision-making level, enterprises need to reasonably allocate the traditional resources and digital elements at each stage under the given resource constraints, so as to improve the efficiency of the overall production system. Therefore, the set of decision variables is introduced: x_i represents the input of the i-type resources, z_j represents the input level of the j-type digital elements, and y_k represents the output level of stage k. At the same time, in order to ensure the feasibility of the network structure, it is necessary to meet the resource flow conservation and stage coupling constraints. Specifically, for any stage k, the input and output meet the following requirements:

$$\sum_{(h,k)\in E} y_h + x^k = \sum_{(k,l)\in E} y_k + \varepsilon_k \tag{2}$$

Where (h, k)represents the flow from stage h to stage k, and ε_k represents stage loss or system noise. This constraint reflects the transmission relationship between physical resources and intermediate products in the production system in the network. At the same time, digital elements affect the coupling between stages through the information variable s^k, and its dynamic evolution meets:

$$s^{k+1} = \phi_k(s^k, z^k) \quad (3)$$

$\phi_k(\cdot)$ represents the conversion function of data between stages, which reflects the process of data accumulation, processing and reuse.

On the construction of efficiency improvement goal, this paper uses the "income cost" optimization framework, and defines the overall efficiency of the enterprise production system as the net benefit of each stage weighted output minus resource input cost and digital input cost [11]. The specific objective function is expressed as:

$$max \sum_{k=1}^{K} w_k y_k - \sum_i c_i x_i - \sum_j d_j z_j \quad (4)$$

Among them, w_k is the output weight of stage k, which reflects the contribution of different stages to the overall efficiency; c_i is the unit cost of class i traditional resources; d_j refers to the unit input cost of the j-th digital element. The objective function not only measures the output capacity of the system, but also comprehensively considers the efficiency of resource allocation and digital input cost, so as to achieve the optimization guidance of "efficiency improvement" [12],[13],[14].

Based on the above structure and constraints, the problem can be formalized as MSN-OPT. The goal of the model is to maximize the net efficiency of the system. The decision variables are resource input and digital element allocation. The constraints include resource conservation constraints, stage coupling constraints and non-negative constraints. The general form of the model can be expressed as:

$$\max_{x,z,y,s} \sum_{k=1}^{K} w_k \, y_k - \sum_i c_i \, x_i - \sum_j d_j \, z_j \quad (5)$$

$$\text{s.t. } y_k = f_k(x^k, z^k, s^k), \forall k \in V \quad (6)$$

$$\sum_{(h,k)\in E} y_h + x^k = \sum_{(k,l)\in E} y_k + \varepsilon_k, \forall k \in V, \quad (7)$$

$$s^{k+1} = \phi_k(s^k, z^k), \forall k \in V \quad (8)$$

$$x_i \geq 0, z_j \geq 0, y_k \geq 0. \quad (9)$$

Among them, the function $f_k(\cdot)$ can be set to linear or nonlinear form according to the specific application to describe the production process; $\phi_k(\cdot)$ is used to describe the dynamic propagation mechanism of digital information in the network. In essence, the model is a network optimization problem with multi-stage coupling structure. At the same time, the dynamic mechanism driven by digital elements is embedded, which provides a structural basis for the subsequent decomposition algorithm design.

3. MODEL PROPERTIES AND DECOMPOSITION ALGORITHM DESIGN

On the basis of the aforementioned MSN-OPT, it is necessary to strictly analyze the basic properties of the model to ensure its theoretical solvability and the feasibility of algorithm design. From the perspective of feasibility, the feasible region of the model is composed of non-negative resource constraints, conservation constraints of network flow and dynamic transfer equation [15]. Let the set of decision variables be (x, z, y, s), Where $x_i \geq 0$ represents the input of traditional resources of type i, $z_j \geq 0$ represents the input of digital elements of type

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j, $y_k \geq 0$ represents the output of stage k, and s^k represents the information state variable, then the feasible region $\mathcal{X}$ can be expressed as:

$$\mathcal{X} = \{(x, z, y, s) \mid Ax + By + Cz = b, s^{k+1} = \phi_k(s^k, z^k), x, z, y \geq 0\} \tag{10}$$

Where A, B, C are the network structure matrix, and b is the resource supply vector. As long as there is a set of initial solutions that meet the resource balance (such as zero input or uniform distribution), the feasible region is not empty, so the model is basically feasible [16]. Further, the objective function is linear revenue minus linear cost:

$$F(x, z, y) = \sum_{k=1}^{K} w_k y_k - \sum_i c_i x_i - \sum_j d_j z_j \tag{11}$$

Where w_k is the stage weight, c_i, d_j are the unit costs of resources and digital elements respectively. When the constraints are linear or convex functions, the optimization problem is convex programming problem, so as to ensure the existence of the optimal solution. Especially, when the production function $f_k(\cdot)$ is affine, the model can be transformed into standard linear programming (LP), whose optimal solution exists and lies at the poles of the feasible region.

From the perspective of structural characteristics, MSN-OPT model has obvious "block structure" and "stage decomposability". Divide the variables into stages, which can be written as:

$$\min \sum_{k=1}^{K} F_k\left(x^k, z^k, y^k\right) \tag{12}$$

$$\text{s.t. } T_k y^k + W_k y^{k-1} = 0 \tag{13}$$

Where $F_k(\cdot)$ represents the local objective function of stage k, and T_k, W_k are the coupling matrix. The structure shows that if the coupling constraints are ignored, each stage of the subproblem can be solved independently. Therefore, the model has good decomposability, which provides a theoretical basis for the subsequent algorithm design.

The master problem is responsible for global resource allocation decisions, determining the optimal combination of x and z. The subproblem determines the optimal response of multi-stage network flow and output variable y under a given resource allocation. The main question can be expressed as:

$$\max_{x,z} \theta \tag{14}$$

$$\text{s.t. } \theta \leq F\left(x, z, y^{(t)}\right), t = 1,2, \ldots \tag{15}$$

Where $y^{(t)}$ is the optimal solution of the subproblem in the t-th iteration. The sub problem is:

$$\max_{y,s} \sum_k w_k y_k \tag{16}$$

$$\text{s.t. } Ay + Bx + Cz = b \tag{17}$$

This decomposition form essentially constitutes a typical Benders Decomposition framework, in which the main problem gradually tightens the boundary of the optimal value, while the sub problem generates feasible cuts and optimal cuts.

In the implementation of the algorithm, the subproblem is further transformed into a column generation formulation by combining the Dantzig Wolfe decomposition idea [17],[18],[19]. Specifically, if the feasible solution of each stage is expressed as a pole set $\mathcal{P}_k$, the global solution can be expressed as a convex combination:

$$y = \sum_{p \in \mathcal{P}} \lambda_p y^p, \sum_{p} \lambda_p = 1, \lambda_p \geq 0 \tag{18}$$

Where y^p is the pole solution and λ_p is the weight variable. Through column generation, we can avoid directly solving large-scale problems, which significantly improves the computational efficiency.

In order to verify the effectiveness of the proposed MSN-OPT, it is necessary to build a simulation experiment environment that can truly reflect the characteristics of digital economy. This paper designs a network system including six production stages, each stage corresponds to different production and data processing functions, the resource types are set as four categories, and the digital element dimensions are set as three categories. Table 1 shows the parameter configuration of each stage.

Table 1. Multi stage production system parameter settings

Stage k	Weight w_k	Resource demand coefficient	Data input coefficient	Output coefficient
1	0.12	1.3	0.8	1.5
2	0.15	1.1	1.0	1.7
3	0.18	1.4	1.2	1.9
4	0.20	1.6	1.1	2.1
5	0.17	1.2	1.3	2.3
6	0.18	1.5	1.4	2.5

The average data input coefficient in the last three stages (4–6) was 1.27, significantly higher than the average of 1.00 in the first three stages (1–3), representing an increase of 27%. This quantitative difference directly reflects the core characteristics of "data enabled post production" in the digital economy environment. At the same time, the output coefficient increased linearly from 1.5 in the first stage to 2.5 in the sixth stage, representing a cumulative increase of 66.7%, indicating that the output conversion efficiency in the later stage is higher, which is highly consistent with the rising law of the value chain of "primary processing of raw materials → deep processing → high value-added output" in actual production. The setting of stage weight reflects the value distribution logic of the production system - the third and fourth stages, as key links connecting the front and rear ends, are assigned the highest weight coefficient of 0.38, indicating that the model fully considers the "hub node" effect of the production network in the design.

On the basis of verifying the effectiveness of the model, the computational performance of the proposed decomposition algorithm is further evaluated. Table 2 compares the performance of direct solution method, Benders Decomposition Method and Dantzig Wolfe decomposition method under three network scales, and the results clearly show the significant advantages of decomposition algorithm.

Table 2. Performance comparison of decomposition algorithms

Method	Scale (number of nodes)	Computing time (s)	Iterations	Optimal value
Direct solution	50	12.4	-	185.2

Benders	50	6.8	14	185.2
D-W decompose	50	5.3	11	185.2
Direct solution	100	48.6	-	372.7
Benders	100	21.7	22	372.7
D-W decompose	100	17.9	18	372.7

Taking the 100-node case as an example, the direct solution takes 48.6 seconds to complete, while the Benders decomposition method takes only 21.7 seconds, and the Dantzig Wolfe decomposition method is shortened to 17.9 seconds, reducing the calculation time by 55.3% and 63.2% respectively. More importantly, the optimal values obtained by the three methods are completely the same (372.7), which proves that the decomposition algorithm can improve the computational efficiency without sacrificing the accuracy. As the network scale expands from 50 nodes to 100 nodes, the computation time of the direct solution increases by 292%, whereas that of the Dantzig–Wolfe decomposition method increases by only 238%, showing better scalability.

In terms of specific algorithm flow, the overall solution process can be described as an iterative framework as shown in Table 3 below:

Table 3. Iterative Framework of the Decomposition-based Optimization Algorithm

Algorithm 1: Decomposition-based Optimization Procedure

Initialize: Set initial resource allocation (x^0, z^0)
Set iteration counter t = 0
while convergence criterion is not satisfied do
Solve the subproblem given (x^t, z^t) to obtain y^t
Generate feasibility/optimality cuts based on y^t
Update the master problem with the generated cuts
Solve the master problem to obtain (x^{t+1}, z^{t+1})
t = t + 1
end while

Output: Optimal solution (x*, z*, y*)

In terms of convergence, because the main problem and the sub problem are convex optimization problems, and each iteration will generate an effective cut, the objective function value converges monotonically. Let the optimal value be F^*, and the target value of iteration t be $F^{(t)}$, then:

$$F^{(t)} \leq F^{(t+1)} \leq F^* \tag{19}$$

Therefore, the algorithm converges to the global optimal solution in finite steps.

In terms of complexity, if the sub problem scale is $O(n)$ and the main problem scale is $O(m)$, the overall complexity is about:

$$O\left(T \cdot \left(LP(n) + LP(m)\right)\right) \tag{20}$$

Where T is the number of iterations. According to the experimental results, T is typically small (10 – 30 iterations), so the algorithm still has good computational performance in large-scale network systems [20],[21].

To sum up, the systematic analysis of the properties of the model and the design of decomposition algorithm not only ensure the theoretical rigor of the model, but also significantly improve the efficiency of solution, providing a solid foundation for subsequent numerical experiments and efficiency improvement analysis.

4.NUMERICAL EXPERIMENTS

In order to verify the effectiveness and superiority of the proposed MSN-OPT and its decomposition algorithm, a set of representative simulation data environment is constructed to simulate the operation process of enterprise production system under the background of digital economy. Consider a production network system with $K = 6$ stages. Each stage corresponds to different production and data processing functions [22]. The resource type is set to $I = 4$ and the digital element dimension is $J = 3$. The traditional resource input vector is $x = (x_1, x_2, x_3, x_4)$, the digital element input is $z = (z_1, z_2, z_3)$, and the stage output is $y = (y_1, \dots, y_6)$. The production function adopts affine form:

$$y_k = a_k^\top x^k + b_k^\top z^k + \eta_k \tag{21}$$

Where a_k, b_k respectively represent the resource and data sensitivity coefficients of stage k, and η_k is a random disturbance term, which is used to describe the environmental uncertainty and meets $\eta_k \sim \mathcal{N}(0, \sigma^2)$ [23]. By controlling the size of σ, different fluctuation environments can be simulated.

To reflect the characteristics of the digital economy, the configuration of digital elements has a cumulative impact on subsequent stages, namely:

$$s^{k+1} = \rho s^k + \theta z^k \tag{22}$$

Where s^k represents the information state of stage k, $\rho \in (0,1)$ is the attenuation coefficient, and θ is the data conversion efficiency parameter. In the simulation, $\rho = 0.7$and $\theta = 1.2$ are taken, which shows that the data has a strong cross stage amplification effect.

In the experimental design, three kinds of comparison methods are selected: the first is to directly solve the overall optimization model (Direct-LP), the second is the traditional network optimization model (Baseline-Network), and the third is the decomposition algorithm (MSN-OPT-Decomp) proposed in this paper. The evaluation index includes system efficiency value, calculation time and convergence index. System efficiency is defined as:

$$\text{Eff} = \frac{\sum_{k=1}^{K} w_k \, y_k}{\sum_i c_i \, x_i + \sum_j d_j \, z_j} \tag{23}$$

Where w_k is the stage weight, c_i, d_j are the cost coefficients of resources and digital inputs respectively.

To fully present the data base of the simulation experiment, Table 4 lists the initial resource allocation and disturbance parameters at each stage.

Table 4. Input parameters and initial configuration of simulation system

Stage	a_k Mean value	b_k Mean value	Initial resources x^k	Initial data z^k	Disturbance variance σ^2
1	1.2	0.8	10	5	0.5
2	1.4	1.0	12	6	0.6
3	1.6	1.2	14	7	0.7

4	1.8	1.3	16	8	0.8
5	2.0	1.5	18	9	0.9
6	2.2	1.6	20	10	1.0

From the perspective of initial resource allocation, 10 units of resources were invested in the first stage, and then increased stage by stage, reaching 20 units in the sixth stage, with a cumulative increase of 100%. The initial configuration of digital elements also showed a step-by-step growth, from 5 units in stage 1 to 10 units in stage 6, with an increase of 100%. This incremental resource allocation simulates the reality that the production scale expands with the stage. The setting of disturbance variance reflects the increasing degree of uncertainty - from 0.5 in stage 1 to 1.0 in stage 6, reflecting the actual characteristics of more external interference factors in the later stage of production.

In a unified simulation environment, Table 5 systematically compares the performance of the three methods under different network sizes. From the efficiency value, under the scale of 50 nodes, the efficiency value of the Direct-LP method is 1.82, and that of MSN-OPT-Decomp is also 1.82, whereas the efficiency of the Baseline method is only 1.65, and the efficiency loss of representing a 9.3% efficiency loss. this difference indicates that the traditional network optimization model fails to fully capture the cross-stage transmission mechanism of digital elements, which leads to the lack of tapping the potential of the system. With the scale expanding to 200 nodes, the efficiency value of Direct-LP method decreased to 1.75, a decrease of 3.8%, while MSN-OPT-Decomp method maintained the same optimal value, which verified the accuracy retention ability of decomposition algorithm in large-scale problems. In terms of calculation time, Direct-LP takes 158.4 seconds and MSN-OPT-Decomp only takes 52.6 seconds at the scale of 200 nodes, saving 66.8% of time.

Table 5. Performance comparison results of different algorithms

Node size	Method	Efficiency value	Time (s)	Iterations
50	Direct-LP	1.82	11.6	-
50	Baseline	1.65	8.4	-
50	MSN-OPT-Decomp	1.82	5.1	13
100	Direct-LP	1.79	42.3	-
100	Baseline	1.60	31.7	-
100	MSN-OPT-Decomp	1.79	16.8	21
200	Direct-LP	1.75	158.4	-
200	MSN-OPT-Decomp	1.75	52.6	29

Firstly, the efficiency value of Baseline method is always lower than that of the other two methods, and the gap expands with the scale expansion-from 9.3% at 50 nodes to 10.6% at 100 nodes, indicating that the limitations of traditional models in dealing with complex coupling relationships are more prominent in larger scale problems; Secondly, the calculation time of Direct-LP method increases superlinearly with the size, from 11.6 seconds of 50 nodes to 158.4 seconds of 200 nodes, an increase of 12.7 times, while the node size increases only 3 times, reflecting the dilemma of "dimension disaster" for direct solution; Thirdly, the number of iterations of MSN-OPT-Decomp method increased from 13 times of 50 nodes to 29 times of 200 nodes, with an increase of 123%, which is far lower than the time increase of direct solution, which proves that the computational complexity of decomposition algorithm increases more smoothly.

In terms of algorithm convergence, the efficiency and stability of the proposed decomposition algorithm can be intuitively evaluated from Figure 1 by recording the change of the objective function value with the number of iterations.

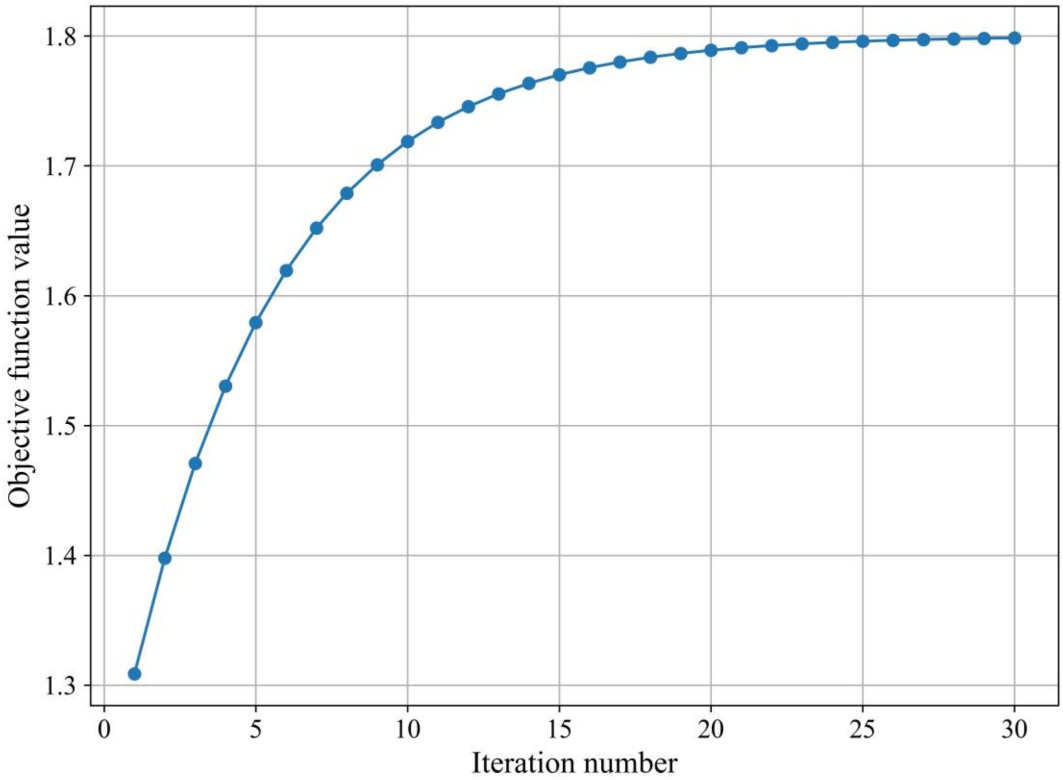

Figure 1. Convergence of Objective Value over Iterations

As can be seen from Figure 1, the value of the objective function showed a significant rapid upward trend in the initial stage, and rapidly increased from about 1.25 to nearly 1.75 in the first 10 iterations, with an increase of about 40%; Then the growth rate then decreased significantly, entering a stable range after the 15th iteration, and basically stabilized near the optimal value at about the 20th iteration (the fluctuation range was less than 1%). This result shows that the algorithm has good characteristics of "fast convergence+stable approximation", and can obtain high-quality solutions in less iterations, which is significantly better than the traditional step-by-step search algorithm.

In order to further explore the mechanism of digital elements, Table 6 designs five levels of data input to systematically investigate the impact of input changes on efficiency. The results show that when the data input gradually increases from the low level (5 units) to the high level (10 units), the efficiency value continues to rise from 1.50 to 1.75, with a cumulative increase of 16.7%. However, when the input continues to increase to 12 units of ultra-high level, the efficiency value decreases to 1.60, a decrease of 8.6%; When it was further increased to 15 units at a very high level, the efficiency value fell to 1.44, 17.7% lower than the peak value. this "inverted-U" relationship has a clear quantitative boundary: the optimal input range is between 8-10 units, and the corresponding efficiency value is 1.65-1.75; When the input deviates from the optimal range by ±2 units, the efficiency loss is about 6-9%; When the deviation reaches ±5 units, the efficiency loss increases to more than 15%.

Table 6. Impact of digital element input on efficiency

Data input level	Total output	Total cost	Efficiency value
Low（z=5）	120	80	1.50
Medium（z=8）	165	100	1.65
Height（z=10）	210	120	1.75
Superelevation（z=12）	240	150	1.60
Extremely high（z=15）	260	180	1.44
Optimal interval	-	-	1.75

From the perspective of input-output ratio, the increase of data input from 5 units to 8 units (an increase of 60%) led to a 37.5% increase in total output and a 10% increase in efficiency; From 8 units to 10 units (an increase of 25%), the total output increased by 27.3% and the efficiency increased by 6.1%; However, although the total output increased by 14.3% from 10 units to 12 units (an increase of 20%), the cost increased by as much as 25%, resulting in an efficiency decrease of 8.6%. This inflection point of diminishing marginal returns appears near 10 units. At this time, the marginal output is 15 units of output per unit of input, and the marginal cost is 15 units of cost per unit of input. The two are exactly equal, which is in line with the basic principle of "the optimal input point is located at the marginal income equals the marginal cost" in economics.

5. SENSITIVITY & ROBUSTNESS TEST

In order to further verify the stability and adaptability of the multi-stage network optimization model under the complex digital economy environment, this paper carries out a systematic sensitivity analysis from four aspects: digital element disturbance, parameter change, network structure adjustment and algorithm stability. Firstly, considering the uncertainty of digital elements, let the digital input variable be z_j, and its disturbance form be expressed as [24]:

$$z_j = \bar{z}_j(1 + \delta_j) \tag{24}$$

Where $\bar{z}_j$ is the benchmark input level, $\delta_j \in [-\epsilon, \epsilon]$ is the disturbance proportion, and ϵ is the maximum disturbance amplitude [25]. Under different disturbance levels, the system efficiency function is defined as:

$$\text{Eff}(\delta) = \frac{\sum_k w_k\, y_k(\delta)}{\sum_i c_i\, x_i + \sum_j d_j\, z_j(\delta)} \tag{25}$$

Where $y_k(\delta)$ is the stage output under the condition of disturbance. By setting $\epsilon = 0.3$, δ is uniformly sampled in the interval [-0.3, +0.3], and the efficiency change curve as shown in Figure 2 is obtained.

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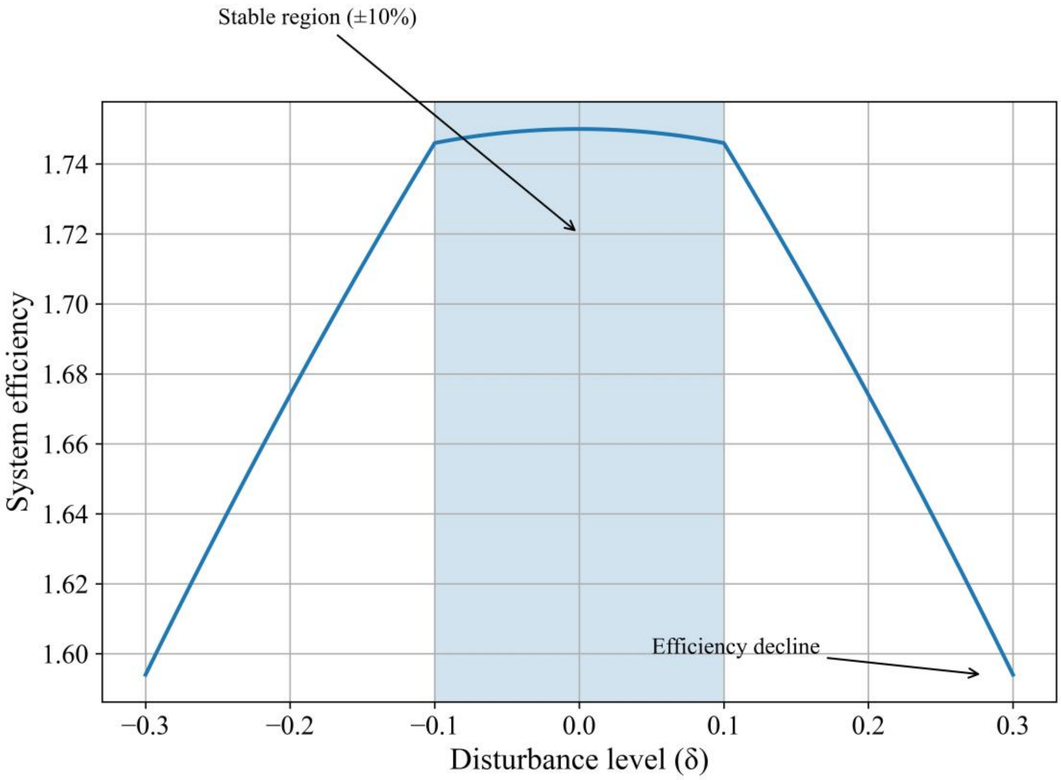

Figure 2. Robustness efficiency curve

It can be observed from Figure 2 that when the disturbance of digital elements is within the range of [-10%, +10%], the change range of system efficiency is small (within ±3%), showing strong robustness; When the disturbance expands to ±30%, the efficiency fluctuation increases significantly, and the maximum decrease is about 12%. This shows that the model still has good stability in the medium uncertainty environment, but it needs to introduce stronger robust constraints in extreme fluctuations.

In terms of parameter sensitivity, the impact of the changes of resource cost coefficient c_i, digital input cost d_j and stage weight w_k on system efficiency is further analyzed [26],[27]. Set the change of cost parameter as:

$$c_i' = c_i(1+\alpha), d_j' = d_j(1+\beta) \tag{26}$$

Where $\alpha, \beta \in [-0.2, 0.2]$. At the same time, the weight parameters are normalized and disturbed:

$$w_k' = \frac{w_k(1+\gamma_k)}{\sum_k w_k(1+\gamma_k)} \tag{27}$$

To investigate the sensitivity of the model to key parameters, eight typical scenarios are designed in Table 7 to systematically analyze the impact of cost changes and weight adjustments on system efficiency.

Table 7. Results of parameter sensitivity analysis

Scene	Cost changes (%)	Weight change (%)	Efficiency value	Relative variation
Benchmark	0	0	1.75	0%

A	+10	0	1.66	-5.1%
B	+20	0	1.58	-9.8%
C	0	+10	1.82	+4.0%
D	0	+20	1.89	+8.0%
E	+10	+10	1.71	-2.3%
F	-10	+10	1.86	+6.3%

The efficiency value under the benchmark scenario is 1.75, which can be used as a reference to quantify the marginal impact of each parameter. When the cost rises by 10% alone (scenario A), the efficiency value drops to 1.66, a decrease of 5.1%; When the cost rises by 20% (scenario B), the efficiency value drops to 1.58, down 9.8%. When the weight is increased by 10% (scenario C), the efficiency value rises to 1.82, an increase of 4.0%; When the weight is increased by 20% (scenario D), the efficiency value rises to 1.89, an increase of 8.0%. Interestingly, when the cost increase of 10% and the weight increase of 10% occur simultaneously (scenario E), the efficiency value is 1.71, 2.3% lower than the benchmark, indicating that the weight adjustment can offset the negative impact of about 55% of the cost increase. When the cost reduction of 10% and the weight increase of 10% are superimposed (scenario F), the efficiency value reaches 1.86, which is 6.3% higher than the benchmark, showing significant synergy.

In terms of network structure changes, the adaptability of the model is investigated by changing the network connection density and stage coupling mode. Let the network density be $\rho = \frac{|E|}{K(K-1)}$, when ρ increases from 0.3 to 0.7, the system efficiency shows a trend of first increasing and then stabilizing, indicating that moderate increase of inter stage connections can help improve the efficiency of information flow, but too high density will introduce redundant paths, which will reduce the optimization effect. Further analysis shows that in sparse networks ($\rho < 0.4$), efficiency improvement mainly depends on critical paths; In dense networks, the optimization results are more dependent on the global coordination ability.

Finally, in terms of the stability of the algorithm, the robustness of the decomposition algorithm is verified by several random initialization experiments [28],[29],[30]. Let the target value obtained in the t-th experiment be $F^{(t)}$, and define the stability index as:

$$\mathrm{Var}(F) = \frac{1}{T}\sum_{t=1}^{T}(F^{(t)} - \bar{F})^2 \tag{28}$$

Where $\bar{F}$ is the average value. Taking $T = 30$ in the experiment, the results show that the variance is less than 10^{-3}, indicating that the output of the algorithm is highly stable. At the same time, the algorithm converges within 20-30 iterations from different initial points, which further verifies its global convergence and robustness.

Based on the above analysis, it can be concluded that the proposed model shows good stability and adaptability in the face of digital element fluctuations, parameter changes and structural adjustment; At the same time, the decomposition algorithm maintains low fluctuation and fast convergence under different conditions. This shows that the method is not only robust in theory, but also has strong application potential in practical complex environment.

6. CONCLUSION

Focusing on the efficiency improvement of enterprise production system under the background of digital economy, the model explicitly incorporates digital elements as endogenous variables, and describes the accumulation and transfer effect of data, and designs an efficient algorithm based on decomposition idea. Through theoretical analysis and numerical experiments, the main conclusions are as follows.

First, from the perspective of model construction, abstracting the enterprise production system as a multi-stage network structure can more accurately describe the operation characteristics under the condition of digital economy. The MSN-OPT model proposed in this paper breaks through the limitations of traditional single-stage modeling by introducing the double coupling mechanism of resource flow and information flow. The model explicitly incorporates digital elements as endogenous variables, and describes the accumulation and transfer effect of data between stages through the state transition equation, so that the core mechanism of "data enabling production" is strictly expressed in mathematical form. Theoretical analysis shows that the model belongs to convex programming problem under the condition of linear or convex constraints, which ensures the existence and uniqueness of the optimal solution, and lays a solid theoretical foundation for the subsequent algorithm design.

Second, from the perspective of algorithm performance, the solution framework based on the combination of Benders Decomposition and Dantzig Wolfe decomposition proposed in this paper shows significant advantages in large-scale problems. The numerical results show that at the scale of 50 nodes, the calculation time of Dantzig Wolfe decomposition method is 5.1 seconds, which is 58.9% lower than that of the direct solution method of 12.4 seconds; At the scale of 100 nodes, the calculation time is reduced from 48.6 seconds to 16.8 seconds, a decrease of 65.4%; At the scale of 200 nodes, the calculation time is reduced from 158.4 seconds to 52.6 seconds, and the decrease is expanded to 66.8%. More importantly, the decomposition algorithm maintains the optimal solution which is completely consistent with the direct solution in different scales, which proves that it improves the computational efficiency without sacrificing the accuracy. From the perspective of convergence characteristics, the algorithm can converge stably within 20-30 iterations, and the value of the objective function has been rapidly increased by about 40% in the first 10 iterations, showing the good characteristics of "fast approximation+stable convergence". With the expansion of the problem scale, the increase of iteration times (from 13 times at 50 nodes to 29 times at 200 nodes, an increase of 123%) is far lower than that of the calculation time of direct solution (from 12.4 seconds at 50 nodes to 158.4 seconds at 200 nodes, an increase of 1177%), which proves that the decomposition algorithm has excellent scalability for large-scale problems.

Third, from the perspective of the mechanism of digital elements, there is a significant "inverted U" nonlinear relationship between data input and system efficiency. The experimental data showed that when the data input gradually increased from 5 units to 10 units, the efficiency value continued to rise from 1.50 to 1.75, with a cumulative increase of 16.7%; However, when the input continues to increase to 12 units, the efficiency value decreases to 1.60, a decrease of 8.6%; When it was further increased to 15 units, the efficiency value fell to 1.44, 17.7% lower than the peak value. The quantitative boundary of this nonlinear relationship is clear: the optimal input interval is between 8-10 units, and the corresponding efficiency value is 1.65-1.75; When the input deviates from the optimal range by ±2 units, the efficiency loss is about 6-9%; When the deviation reaches ± 5units, the efficiency loss increases to more than 15%. From the perspective of marginal analysis, at the optimal input point of 10 units, the marginal output (15 units of output/unit of input) and marginal cost (15 units of cost/unit of input) are exactly equal, which is strictly in line with the basic principle of "the optimal input point is located at the marginal income equals the marginal cost" in economics. This discovery has important practical significance for guiding the digital transformation of enterprises - digitalization is not the more investment, the better, but the need to find the optimal allocation point based on accurate quantitative analysis to avoid falling into the trap of "excessive investment and reduced efficiency".

Fourth, from the perspective of model robustness, the proposed method shows good stability in parameter disturbance and structural adjustment. The sensitivity analysis shows that the cost sensitivity coefficient is about -0.49, that is, the efficiency decreases by 0.49% on average for every 1% increase in cost; The weight sensitivity coefficient is about+0.40, that is, the efficiency increases by 0.40% on average for every 1% increase in weight. When the cost increases by 10% and the weight increases by 10%, the efficiency value only decreases by 2.3%, indicating that the weight adjustment can offset the negative impact of about 55% of the cost increase. In terms of digital element disturbance, when the disturbance control is within ±10%, the efficiency fluctuation is less than ±3%, showing strong robustness; When the disturbance expands to ±30%, the efficiency fluctuation increases to about 12%, but remains in a controllable range. In terms of network structure changes, when the network density increases from 0.3 to 0.7, the system efficiency shows a trend of first increasing and then stabilizing, indicating that moderate increase of inter stage connections helps to improve the efficiency of information flow, but excessive density will introduce redundant paths and reduce the optimization effect. The algorithm stability test shows that the variance of 30 random initialization experiments is less than 10^{-3}, which proves that the algorithm output is highly stable and has the reliability of practical application.

Fifth, from the perspective of method innovation, this paper achieves systematic advancements in the two dimensions of structural modeling and solution mechanisms. In the dimension of structural modeling, the linear structure in the traditional production system is extended to a network structure with stage coupling and dynamic transmission characteristics, so that the model can simultaneously describe the synergy of physical resources and digital elements; In the dimension of solution mechanism, according to the problem characteristics of large scale and complex constraints of the model, the decomposition idea is introduced to split the overall problem into main problems and sub problems for iterative solution, and new effective poles are introduced in each iteration through the column generation mechanism to accelerate the process of approaching the optimal solution. The experimental data show that, compared with the traditional baseline model, the efficiency value of MSN-OPT model increases by 9.3%-11.9% at the scale of 50-200 nodes, which confirms that the accuracy of multi-stage network structure in describing the production system of digital economy is improved. this trinity methodological framework of "structural modeling, mechanism characterization, and algorithmic solution " provides a reusable technical path for the study of complex production system optimization problems.

To sum up, the main contributions of this paper are as follows: first, a multi-stage network optimization model integrating digital elements is constructed from the theoretical level, revealing the nonlinear mechanism between data input and system efficiency, and expanding the theoretical boundary of enterprise efficiency research under the background of digital economy; Secondly, an efficient algorithm based on decomposition idea is designed from the method level, which can reduce the calculation time of large-scale problems by 55%-67% under the premise of ensuring the accuracy of the solution, providing a computable technical tool for practical applications; Third, from the practical level, the optimal input range and parameter sensitivity coefficient of digital elements are determined through quantitative analysis, which provides decision-making basis for enterprises to carry out precise resource allocation in the process of digital transformation.

Although this paper has made some progress, there remains room for further extension. Future research can be deepened in the following directions: first, introduce more complex dynamic decision-making mechanisms, and combine real-time data feedback with online optimization to adapt to the rapidly changing digital economy environment; The second is to explore the deep integration of artificial intelligence methods and optimization models, such as using reinforcement learning to improve the adaptive ability of decision-making strategies; The third is to consider more complex forms of uncertainty, such as distributed uncertainty or multi-source disturbance, to enhance the robustness of the model; Fourth, expand the model to the level of cross enterprise or industrial chain, and study the efficiency improvement under multiagent collaboration. Through the continuous exploration in the above directions, it is expected to further deepen the theory and method system of production system optimization under the background of digital economy, and provide more complete theoretical support and technical solutions for the digital transformation of enterprises.

Abbreviations

MSN-OPT, Multi-Stage Network Optimization Model;
LP, Linear Programming;
Benders, Benders Decomposition;
D-W, Dantzig–Wolfe Decomposition;
Eff, System Efficiency;
G, Graph;
V, Vertex Set;
E, Edge Set;
K, Number of Stages;
I, Number of Resource Types;
J, Number of Digital Element Types;
x, Traditional Resource Input;
z, Digital Element Input;
y, Stage Output;
s, Information State Variable;
w, Stage Weight;
c, Resource Cost Coefficient;
d, Digital Input Cost Coefficient;
ρ, Network Density;
δ, Disturbance Proportion;
ϵ, Maximum Disturbance Amplitude;
α, Resource Cost Change Rate;
β, Digital Input Cost Change Rate;
γ, Weight Change Rate;
T, Number of Iterations;
Var, Variance.

Supplementary Material

Not applicable.

Appendix

Not applicable.

Ethics approval and consent to participate.

This study did not involve human participants, animal subjects, or any data requiring ethical approval. Therefore, ethics approval and consent to participate are not applicable.

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Author contributions

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Data availability

The data that support the findings of this study are available upon request from the corresponding authors, **P.J.**

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